# A Cellular Automata Model for Soil Erosion by Water

**D. D'Ambrosio[1], S. Di Gregorio[1], S. Gabriele[2], R. Gaudio[2]**

[1]Department of Mathematics, University of Calabria, 87036 Arcavacata di Rende (CS), Italy
[2]Istituto di Ricerca per la Protezione Idrogeologica nell'Italia Meridionale e Insulare – Consiglio Nazionale delle Ricerche (CNR-IRPI) – Via Cavour, 87030 Rende (Cosenza), Italy

Name of the author to whom offset requests should be sent: Roberto Gaudio.

**Abstract.** A Cellular Automata model for soil erosion by water, SCAVATU, was developed. It involves a larger number of states in comparison to the previous models: altitude, vegetation density, water depth, water run-up height, infiltration, outflow towards neighbouring cells, inflows, eroded material, sediment transport, deposited material, sediment transport fluxes.

It was applied to the Armaconi basin, Calabria, Italy, considering some of the most significant processes of the phenomenon: water flow, infiltration, soil erosion by water flow, sediment transport and deposition. Simulations gave encouraging results, in agreement with the findings of other studies.

## 1 Introduction

Complex phenomena, the behaviour of which can be described in terms of local interactions of their constituent parts, can be frequently modelled efficiently by novel methods inspired to Parallel Computing models (Di Gregorio et al., 1996). Cellular Automata (CA) represent this kind of models. They are based on a regular division of space in cells, each one embedding identical finite automata (fa), the input of which is given by the states of the neighbouring cells; fa have an identical transition function, which is simultaneously applied to each cell. At the time t=0, fa are in arbitrary states, representing the initial conditions of the system, then the CA evolves by changing the state of all fa simultaneously at discrete times, according to the transition function of the fa.

Applications of CA are very broad; they range from microscopic simulations of physical and biological phenomena (Gutowitz, 1991) to macroscopic simulations of geological and social processes (Weimar, 1995).

Soil erosion is a potential CA application field, because it can be considered as a system evolving exclusively by means of local interactions. The complexity of the problem generates systems of differential equations, which cannot be easily solved without making substantial simplifications. In order to overcome these difficulties, the CA approach has been tried in the past.

*Correspondence to*: Roberto Gaudio

Smith (1991) compares the Huyghens wavefront method with a CA method for the erosion of landforms, obtaining a good agreement for two-dimensional instances. Murray and Paola (1994, 1997) developed an interesting cellular braided stream model with significant qualitative success in simulating the main features of the phenomenon. Pilotti and Menduni (1997) used two-dimensional Lattice Gas Automata for sediment erosion and transport due to laminar sheetflow.

We developed a CA model for soil erosion, following an empirical method for modelling and simulating macroscopic phenomena (Di Gregorio and Serra, 1999). It involves a larger number of states in comparison to the previous models. The main features of the method are the following: each characteristic, relevant to the evolution of the system and relative to the space portion corresponding to the cell, is identified as a component of the state (a substate). The values associated with the substates can vary depending on the interactions among substates inside the cell (internal transformation) and local interactions among cells. Local interactions are treated in terms of flows of a quantity (substate) towards the neighbouring cells, in order to achieve equilibrium conditions.

## 2 The CA model for the simulation of soil erosion

The following CA model for soil erosion by water, SCAVATU (**S**imulation by **C**ellular **A**utomata for the Erosion of **VA**st **T**erritorial **U**nits - to be read "ska:´va:tu"; the acronym has been devised to mean "eroded" in Calabrian and Sicilian dialects, even if the model can be applied to vast or small areas), can be seen as a two-dimensional plane, partitioned in square cells of uniform size:

$$SCAVATU = \langle R, X, S, P, \sigma, \gamma \rangle \quad (1),$$

where:

- $R = \{(x, y)|\ x, y \in N, 0 \le x \le l_x, 0 \le y \le l_y\}$ is the set of points with integer co-ordinates in the finite region, where the phenomenon evolves. N is the set of natural numbers; $l_x$ and $l_y$ represent the limits of the region;
- the set X = {(0,0), (0,1), (0,-1), (1,0), (-1,0)} identifies the von Neumann neighbourhood, which influences the change in state of the central cell (Fig. 1);
- the finite set S of the states of the fa is the Cartesian product of the sets of substates, described in Table 1:

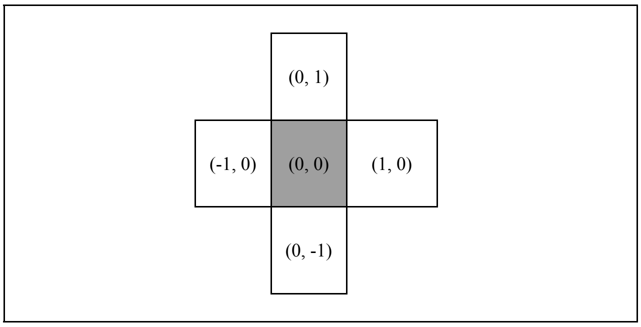



**Fig. 1.** Neighbourhood of von Neumann

$S = S_z \times S_{vd} \times S_{wd} \times S_r \times S_E \times S_t \times S_d \times S_I \times S_O \times S_{tf}$ (2);

**Table 1.** Substates of the SCAVATU model

| Substate | Description |
|---|---|
| $S_z$ | altitude |
| $S_{vd}$ | vegetation density |
| $S_{wd}$ | water depth |
| $S_r$ | water run-up height (water depth plus kinetic head, see §3.4) |
| $S_E$ | eroded material |
| $S_t$ | sediment transport |
| $S_d$ | deposited material |
| $S_I = S_{I1} \times S_{I2} \times S_{I3} \times S_{I4}$ | product of the substates regulating infiltration (see §3.1) |
| $S_O$ | outflow towards neighbouring cells (inflows $S_i$ are trivially derived) |
| $S_{tf}$ | sediment transport fluxes |

- P is the finite set of global parameters of the CA (Table 2), constant in time and space, which effect the transition function:

$P = \{ p_\delta, p_{ts}, p_{TOl}, p_{TOu}, p_{Emax}, p_{Tr}, p_{Tvd}, p_{Trm}, p_{tc}, p_f, p_{Rr}, p_{tmax} \}$ (3).

**Table 2.** Parameters of the SCAVATU model

| Parameter | Description |
|---|---|
| $p_\delta$ | side size of the CA cell |
| $p_{ts}$ | CA time step |
| $p_{TOl}$ | lower outflow threshold |
| $p_{TOu}$ | upper outflow threshold |
| $p_{Emax}$ | cell maximum eroded soil in a step |
| $p_{Tr}$ | run-up normalisation threshold |
| $p_{Tvd}$ | veg. density normalisation thresh. |
| $p_{Trm}$ | run-up threshold of motion |
| $p_{tc}$ | transport capacity (as % of $w_d$) |
| $p_f$ | head loss due to friction in a step |
| $p_{Rr}$ | water flow relaxation rate |
| $p_{tmax}$ | max cell sedim. transport in a step |

- $\sigma: S^5 \rightarrow S$ is the deterministic state transition. It is specified by three internal transformations (T1, T2, T3) and two local interactions (I1, I2):

T1) infiltration: $\sigma_{T1}: S_I \times S_{wd} \rightarrow S_I \times S_{wd}$;

T2) soil erosion by water flow:

$\sigma_{T2}: S_r \times S_E \times S_{wd} \times S_{vd} \rightarrow S_E \times S_d$;

T3) distribution of cell eroded soil in sediment transport and deposition: $\sigma_{T3}: S_r \times S_{wd} \times S_d \times S_t \rightarrow S_d \times S_t$;

I1) water flows and sediment transport:

$\sigma_{I1}: (S_z \times S_r \times S_{wd} \times S_t)^5 \rightarrow (S_O \times S_{sf})^4$;

I2) run-up determination: $\sigma_{I2}: (S_r \times S_z)^5 \times S_i^4 \rightarrow S_r$;

- $\gamma: N \rightarrow S_{wd}$ specifies the variation of water depth in cells due to rain at each CA step, $t_s \in N$; it represents the rainfall history.

## 3 Specification of the transition function

The transition function has been specified with long programming subroutines. For the sake of simplicity, we illustrate the main ideas underlying the fa state transition using qualitative and semi-qualitative statements.

### 3.1 Infiltration (T1)

The height of water present in a cell, generated by rainfall and by the balance between inflows and outflows among the cell and its neighbourhood (see §3.4), is called water depth, $w_d$. Part of it infiltrates, depending on the degree of saturation of the soil, i.e. on the water content.

Infiltration, I, is computed according to the following model of soil. The soil under the cell can be considered as a water reservoir of given capacity, C (Fig. 2).

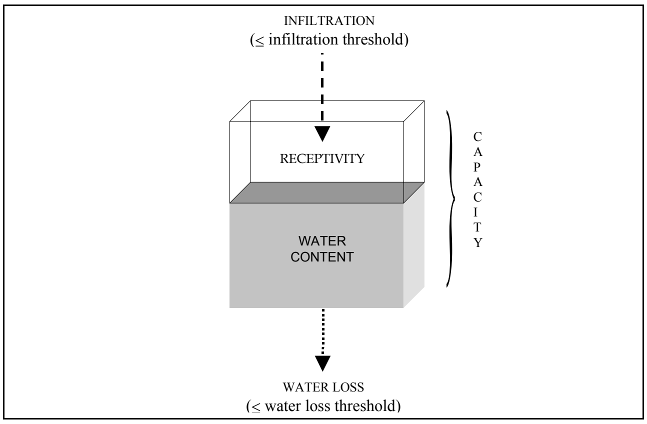



**Fig. 2.** Schematic of infiltration process

In a CA step it can receive a maximum water quantity, $T_I$ (infiltration threshold), and can lose a maximum water quantity, $T_L$ (water loss threshold). The water content is called $w_c$. The difference between C and $w_c$ gives the cell receptivity, rc. $S_{I1}$, $S_{I2}$, $S_{I3}$ and $S_{I4}$ correspond respectively to C, $T_I$, $T_L$ and $w_c$.

Water loss, $w_L$, is the minimum value between $w_c$ and $T_L$. Analogously, infiltration is the minimum value among $w_d$, $T_I$ and rc. The variations of water depth, water content and receptivity are respectively:

$\Delta w_d = -I$ (4);

$\Delta w_c = I - w_L$ (5);

$\Delta rc = -\Delta w_c$ (6).

Soil hydraulic conductivity is taken as constant. The model does not account for variations depending on the degree of saturation of the soil.

### 3.2 Soil erosion by water flow (T2)

Soil erosion, E, is supposed depending mainly on the total outflow, O, from the cell to the neighbourhood. If it is lower than a lower outflow threshold, $T_{Ol}$, then there is no erosion; if it is larger than an upper outflow threshold, $T_{Ou}$, then the maximum erosion, $E_{max}$, occurs. In intermediate cases, the contribution of both water run-up (water depth plus kinetic head, see §3.4), r, and vegetation density, $v_d$, in the cell is considered, according to a logistic-like curve, that uses normalised values of outflow, $O_n$, run-up, $r_n$, and vegetation density, $v_{dn}$ (Fig. 3).

The normalisation is limited by upper thresholds for each contribution, i.e. if run-up and vegetation density are greater than their respective thresholds, $T_r$ and $T_{vd}$, then they assume a maximum value, equal to $\pi/2$. Figure 4 shows the Pascal-like procedure for erosion computation.

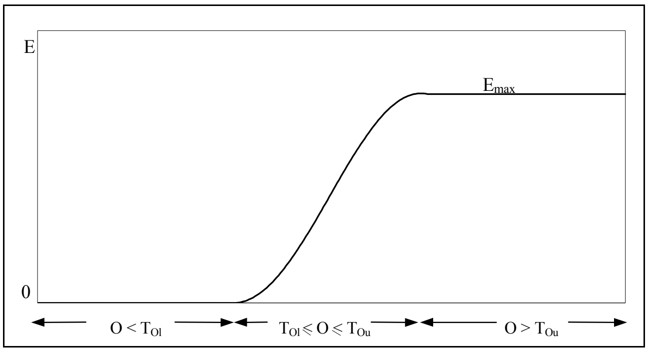



**Fig. 3.** Logistic-like curve: $E = E_{max} \cdot \sin(O_n) \cdot \sin(r_n) \cdot \cos(v_{dn})$ per $T_{Ol} \le O \le T_{Ou}$

### 3.3 Distribution of cell eroded soil in sediment transport and deposition (T3)

The sediment present in a cell, s, is the sum of three terms: 1) the eroded material in the cell itself, E; 2) the sediment previously deposited, d; 3) the sediment fluxes from the neighbourhood, i.e. the total transport t. The available sediment in the cell, s, can be partially or totally transported, or stay in a condition of deposition.

The solid material is deposited when run-up is lower than an opportune threshold of motion, $T_{rm}$, and transported when it is greater than $T_{rm}$. The sediment load cannot exceed the transport capacity, $t_c$. Figure 5 shows the Pascal-like procedure for transport/deposition computations. "New" indicates the new value of variables.

```
Procedure erosion; ……
begin
  if O > TOu then
    E := Emax
  else
    if O < TOl then
      E := 0
    else
      begin
        On := (O / TOu) * π/2;
        if r > Tr then
          rn := π/2
        else
          rn := (r / Tr) * π/2;
        if vd > Tvd then
          vdn := π/2
        else
          vdn := (vd / Tvd) * π/2;
        E := Emax *sin(On) * sin(rn) * cos(vdn);
      end
end;
```

**Fig. 4.** Pascal-like procedure for erosion computation

```
Procedure transport_deposition;…………
begin
  s := E + d + t;
  if r < Trm then
    begin
      d_new := s;
      t_new := 0
    end
  else
    if s < tc then
      begin
        d_new := 0;
        t_new := s
      end
    else
      begin
        t_new := tc;
        d_new := s - tc
      end
end;
```

**Fig. 5.** Pascal-like procedure for transport/deposition computations

### 3.4 Water flows and sediment transport (I1)

The total head in a cell, H, includes altitude, water depth and kinetic head (Fig. 6). Run-up, r, represents the water depth virtually increased due to kinetic head. The present version of SCAVATU does not account for altitude variations due to erosion/deposition phenomena (negligible with respect to the initial values), therefore run-up indicates the variable part of the total head (Fig. 6).

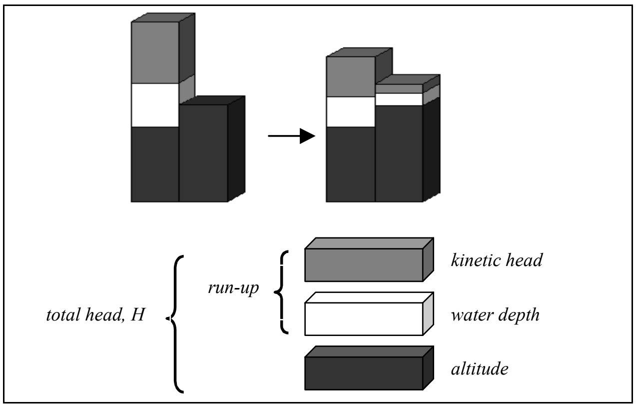



**Fig. 6.** Definition of total head and run-up. Example of minimisation

A dynamical system evolves towards equilibrium conditions. The law of state change, which rules water flows (energy exchanges) from the central cell to its neighbours, is based on the minimisation of the differences of the total head in the neighbourhood. In the present case only the variable part of H, that is r, can be considered to minimise the differences.

A sketch of the minimisation algorithm (Di Gregorio and Serra, 1999) follows:

(a) let $E_i$ indicate the altitude of the central cell for i=0 and the total heads of its neighbours for i=1,2,3,4; cells with $E_i$>H[0] are eliminated;

(b) for the set of non-eliminated cells, A, the following average value is computed:

$$\overline{E} = \left( r[0] + \sum_{i \in A} E_i \right) / (\#A) \qquad (7);$$

(c) cells with $E_i > \overline{E}$ are eliminated;

(d) go to step (b) until no cell is eliminated;

(e) r[0] is distributed among non-eliminated cells, so that $H_i = \overline{E}$ for $i \in A$.

An example of minimisation of differences is shown in Fig. 7. The initial conditions are: H[0]=4, E[0]=z[0]=3, r[0]=11, E[1]=H[1]=20, E[2]=H[2]=6, E[3]=H[3]=18, E[4]=H[4]=11.

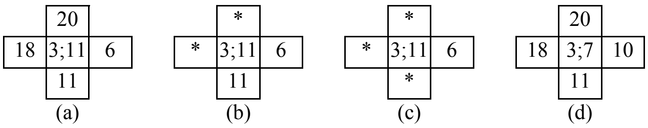



**Fig. 7.** Example of minimisation: (a) E[1] > H[0], E[3] > H[0] ⇒ cells 1 and 3 will be eliminated; (b) $\overline{E}$ = 31/3 =10.3, E[4] > $\overline{E}$ ⇒ cell 4 will be eliminated; (c) $\overline{E}$ = 20/2 = 10; no cell will be eliminated and the remaining cells will assume the value of $\overline{E}$; (d) configuration after the minimisation: ΔH[2] = (10 – 6) = 4, ΔH[1] = ΔH[3] = ΔH[4] = 0.

The outflow from the central cell to j-th neighbour, $O_j$, is given by the following expression:

$$O_j = \left( w_d[0] \cdot \frac{\Delta H_j}{r[0]} \right) \cdot R_r \qquad (8),$$

which indicates that the water depth of the central cell, $w_d[0]$, is distributed towards the neighbouring cells proportionally to the run-up aliquotes determined with the minimisation algorithm, $\Delta H_j$ being the difference between the total head values after and before the minimisation. Besides, a 'relaxation rate', $0<R_r<1$, is introduced to take into account that water distribution is not effected in a single CA step.

The sediment fluxes, t, are proportional to the relative water flows, $O_j$.

### 3.5 Run-up determination (I2)

The water depth in the central cell at the next step, $w_{dnew}[0]$, is computed as follows:

$$w_{dnew}[0] = w_d[0] - O + \sum_{i=1}^{4} q_i = w_{dr}[0] + \sum_{i=1}^{4} q_i \qquad (9),$$

where $w_d[0]$ is the initial water depth, O is the total outflow towards the neighbourhood, $w_{dr}[0]$ is the residual water and $q_i$ (i=1,2,3,4) is the inflow from the i-th neighbouring cell.

The new run-up is computed as the weighted average of the total heads on the relative water depths, $w_{dr}$ and $q_i$, minus the altitude of the central cell, z, and the head losses due to friction, f. If $r_{new} < w_{dnew}$, then the value of $w_{dnew}$ is imposed:

$$r_{new} = \max\left( \frac{w_{dr} H + \sum_{i=1}^{4} (q_i H_i)}{w_{dr} + \sum_{i=1}^{4} q_i} - z - f ; w_{dnew} \right) \qquad (10).$$

Figure 6 illustrates qualitatively a possible situation.

## 4 The Armaconi catchment

The CA model was tested in the catchment of the Fiumara Armaconi, a tributary of the River Amendolea (Calabria, Southern Italy; Fig. 8).

The Armaconi basin is located on the Ionian side of the Southern part of the Calabria region. It has the following characteristics: elevations 116.4 to 670 m above the mean sea water level, average altitude 367.3 m, surface area 1.8 $km^2$ (Fig. 9a). The bedrock geology consists of Paleozoic crystalline rocks to Tertiary clastic sedimentary rocks.

Water erosion in the Armaconi basin was investigated by Gabriele et al. (1999), who adopted a revised version of the well-known Universal Soil Loss Equation (USLE) by Wischmeier and Smith (1978). Gabriele et al. followed a distributed approach (Pilotti and Bacchi, 1997), dividing the catchment area in several morphological units (homogeneous cells) and applying the revised USLE to each cell. In particular, the rainfall factor was determined through the analysis of all of the 185 rainfall erosive events occurred during 24 years of observation, in the period from 1971 to 1997. Vegetal coverage was represented using the Normalised Difference Vegetation Index (NDVI), which provides information about its 'density' (Kriegler et al., 1969; Rouse et al., 1973). The NDVI values range from –1.0 to 1.0; negative/positive values represent bare/covered areas. The NDVI map (Fig. 9b) was obtained from a Landsat scenario of July 1995; light/dark cells represent nude/vegetated areas. It is evident the scarce presence of vegetation in the Armaconi basin.

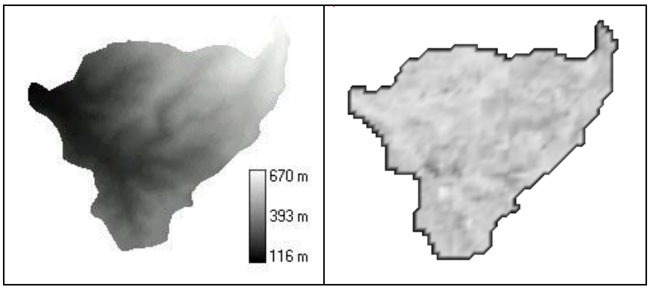



**Fig. 9.** Armaconi basin. (a) Digital Elevation Model; (b) NDVI map; light/dark cells represent nude/vegetated areas.

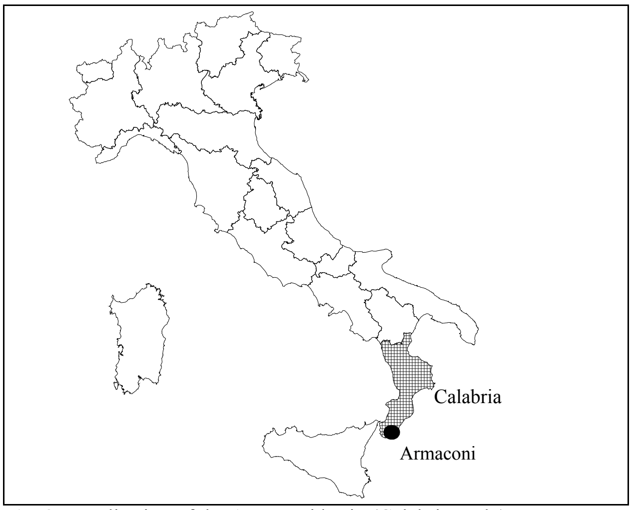



**Fig. 8.** Localisation of the Armaconi basin (Calabria, Italy)

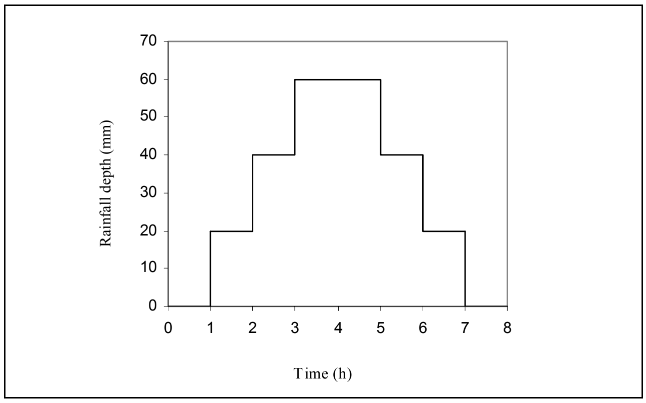



**Fig. 10.** Simulation pluviogram

The resulting average erosion rate of approximately 1 mm/y was in agreement with the eroded volumes determined through the comparison of two aerial photographs of the catchment, respectively dating back to 1972 and 1997.

## 5 First applications of the CA model to the Armaconi basin

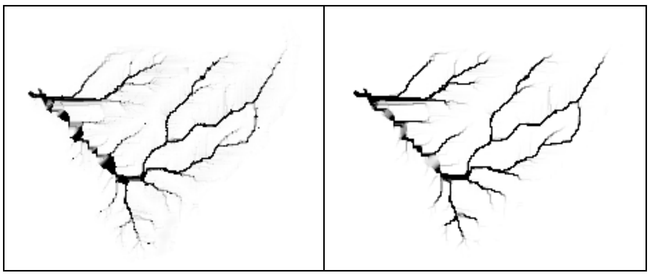
Many SCAVATU simulations were performed considering the model in a simplified context, where some substates ($S_E$ and $S_I$) assume average values in the whole area of the Armaconi basin. An artificial pluviogram, simulating an episode of heavy rain, was devised (Fig. 10). Table 3 shows the simulation parameters and their set input values.

Results at different CA steps are shown in Figs. 11 to 13, where water depths, erosion, sediment transport and deposition are localised in the basin and represented with a grey scale, from 0 μm (white) to 255 μm or more (black).

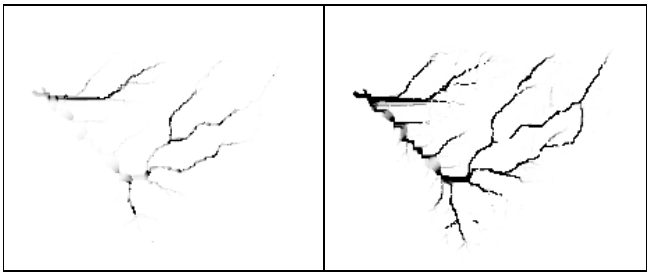
Revisore: Comment to results; calibration of the model (values of the parameters); comparison with USLE results.

**Table 3.** Simulation values of parameters

| | |
|---|---|
| Cell side size ($p_\delta$) | 10 m |
| CA time step ($p_{ts}$) | 3 s |
| Lower outflow threshold ($p_{TOl}$) | |
| Upper outflow threshold ($p_{TOu}$) | |
| Cell maximum eroded soil in a step ($p_{Emax}$) | 25 μm |
| Run-up normalisation threshold ($p_{Tr}$) | |
| Vegetation density normalisation threshold ($p_{Tvd}$) | |
| Run-up threshold of motion ($p_{Trm}$) | |
| Transport capacity as a percentage of water depth ($p_{tc}$) | 20% |
| Head loss due to friction in a step ($p_f$) | 0.5 m |
| Water flow relaxation rate ($p_{Rr}$) | 0.09 |
| Maximum sediment transport in a cell in a step ($p_{tmax}$) | 25 μm |

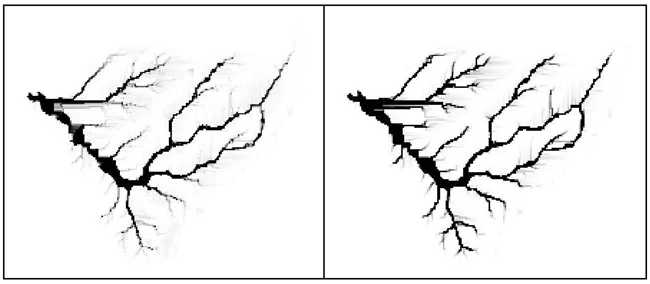


(a) (b)

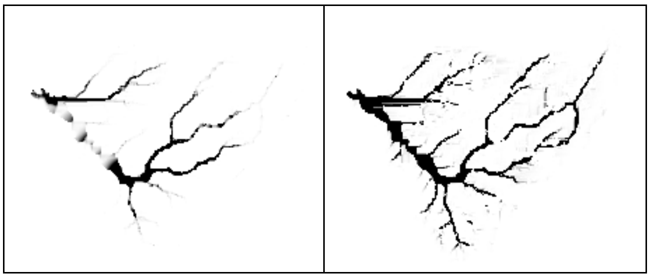


(c) (d)

**Fig. 11.** Results of a SCAVATU simulation for the Armaconi basin after 2 h (rainfall height 40 mm). Grey scale ranges from 0 µm (white) to 255 µm or more (black). (a) water depths; (b) erosion (241.4 $m^3$); (c) sediment transport (13.5 $m^3$); (d) deposition (227.4 $m^3$).

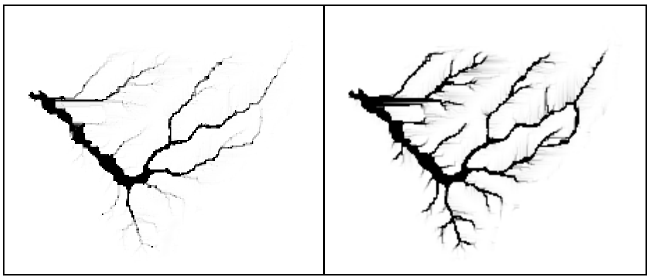


(a) (b)

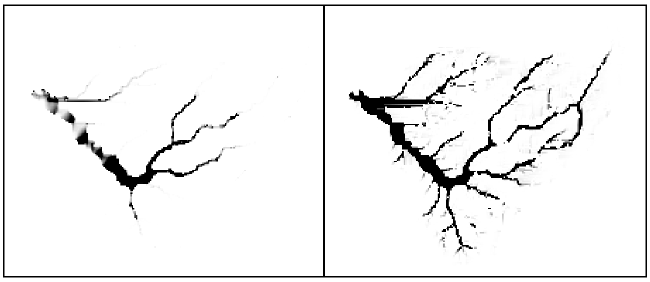


(c) (d)

**Fig. 12.** Results of a SCAVATU simulation for the Armaconi basin after 4 h (rainfall height 60 mm). Grey scale ranges from 0 µm (white) to 255 µm or more (black). (a) water depths; (b) erosion (2155.1 $m^3$); (c) sediment transport (94.3 $m^3$); (d) deposition (2046.2 $m^3$).



(a) (b)



(c) (d)

**Fig. 13.** Results of a SCAVATU simulation for the Armaconi basin after 6 h (rainfall height 20 mm). Grey scale ranges from 0 µm (white) to 255 µm or more (black). (a) water depths; (b) erosion (4546.7 $m^3$); (c) sediment transport (161.4 $m^3$); (d) deposition (4350.8 $m^3$).

## 5 Conclusions

Initial model results are encouraging as they show that: a) the volumes of erosion correspond adequately to the rainfall duration and intensity, in agreement also with the results of other studies (Gabriele et al., 1999); b) most areas of greater erosion are identified. Future research by the authors will consider the following internal transformation and local interactions: 1) soil erosion by direct rainfall; 2) altitude decrease [Revisore: A few more words are needed to make clear what aspects of soil erosion the altitude decrease is related to (rainfall?)]. The model can be applied for the simulation of the possible effects of natural events or river management works [Revisore: How does the soil loss model relate to river management? The rules for rivers would be expected to be quite different than for hillslopes] (e.g., occlusion of a natural channel, construction of check dams or embankments, etc.).

*Acknowledgement*. The authors are grateful to the reviewers for their useful remarks.

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